2/1/2020

UNIT-3

Losses of Prestressing

Pretension:

1. Elastic deformation
2. Shrinkage of concrete.
3. Creep of concrete.

Post tensioned:

1. No elastic deformation due to simutaneously tensioned & sucessive tensioned elastic deformation occur.
2. Shrinkage of concrete
3. Creep of concrete.
4. Friction.
5. Anchorage slip.

Losses due to elastic deformation:

$$E_e = \frac{f_c}{E_c} \; ; \quad \text{strain} = \frac{\text{force (stress)}}{\text{young's modolus}}$$

$$= \frac{f_c}{E_c} E_s$$

Es = young's modolus of steel

Ec = young's modolus of concrete

fc = Prestressing force.

modular ratio = $\left[\frac{E_s}{E_c}\right] f_c$

Loss due to elastic deformation $= \alpha_e f_c$.

Losses due to shrinkage: It is due to shortening of wires.

Prestressing tensioned $= 300 \times 10^{-6}$

Post tensioned $= \dfrac{200 \times 10^{-6}}{\log_{10}(T+2)}$

$\Big\}$ ε_{cs}. (Pg: 16)

Loss due to shrinkage $= \varepsilon_{cs} \times E_s$

4.

3/11/2020

Loss due to creep:

Deformation due to sustained load is called as creep.

1) Ultimate creep strain method $= \varepsilon_{cc} * f_c \, E_s$.

2) Creep coefficient method $(\phi) = \dfrac{\text{creep strain}}{\text{Elastic strain}} = \dfrac{\varepsilon_c}{\varepsilon_e}$

$$\varepsilon_c = \phi . \varepsilon_e$$

$$\varepsilon_c = \phi \left(\frac{f_c}{E_c} \right) E_s$$

$$\varepsilon_c = \phi \left(\frac{E_s}{E_c} \right) f_c$$

$$= \phi \, \alpha_e \, f_c$$

Loss due relaxation of steel:

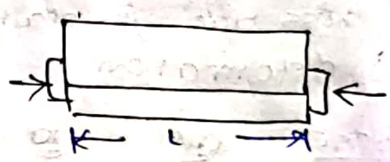
decrease stress with time under constant strain

$\underbrace{0.5 f_{pu} \text{ to } 0.85 f_{pu}}_{\text{Initial stresses}}$. varies $\underbrace{0 - 90 \ N/mm^2}_{\text{Relaxation losses}}$.

Loss due to Anchorage slip: (Post tension)

$$\Delta = \frac{PL}{A \, E_s}$$

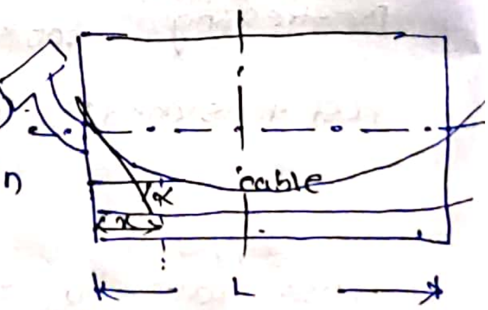
$$\frac{P}{A} = \frac{E_s \Delta}{L}$$



Loss due to friction: (Post tension)

$$P_x = P_0 \, e^{-(\mu \alpha + k x)}$$

(Pg: 33)



where

μ = coefficient friction in curve.

k = coefficient for wave effect.

α = cumulative angle.

P_0 = Prestressing force at the tensioning end.

Elastic deformation problems:

1. A Pre tensioned concrete beam of rectangular c/s section 150mm wide & 300mm deep is Pre-stressed by 8 tensile wires of 7mm ϕ are located at 100mm soffit of the beam. If the wires are tensioned to a Stress of $1100 N/mm^2$ calculate the percentage loss of stress due to elastic deformation. Assume the modulus of elasticity of concrete (E_c) and steel (E_c) as 31.5 & 210 N/mm^2

Sol: Given data.

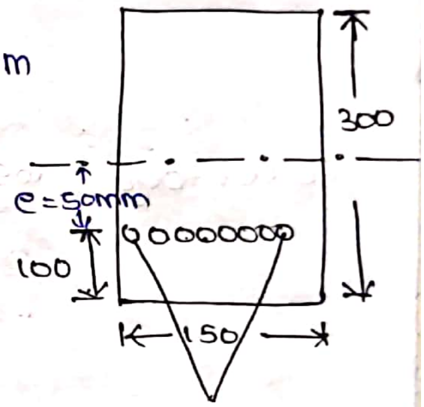
c/s section = 150mm * 300mm

c/s Area = $45000 mm^2$

Stress = $1100 N/mm^2$

E_c = $31.5 N/mm^2$

E_s = $810 N/mm^2$



$$\text{force}(F) = \frac{\text{Load}(P)}{\text{Area}(A)}$$

Stress (F) * Area (A) = Load(P)

$1100 * 45 * 10^3$

$$\text{Area of steel bars} = n \cdot \frac{\pi}{4}(d)^2$$

$$= 8 \cdot \frac{\pi}{4}(7)^2$$

$$= 307.87 \ mm^2$$

$$\text{Load}(P) = 1100 * 307.87.$$

$$P = 338.66 * 10^3 \ kN$$

$$\text{Stress at level of steel } (f_c) = \frac{P}{A} + \frac{Pe^2}{I}$$

f_c = 338.66 .

$$I = \frac{bd^3}{12} = \frac{(150)(300)^3}{12}$$

$$I = 337.5 * 10^6 \ mm^4$$

$$f_c = \frac{338.66 \times 10^3}{45 \times 10^3} + \frac{338.66 \times 10^3 \times (50)^2}{337.5 \times 10^6}$$

$$= 7.52 + 2.5$$

$$\boxed{f_c = 10 \ N/mm^2}$$

Loss of stress due to elastic deformation of concrete $= \alpha_e f_c$

$$= \left(\frac{E_s}{E_c}\right) f_c$$

$$= \left(\frac{210}{31.5}\right) \times 10$$

$$= 66.66 \ N/mm^2$$

Percentage loss of stress in steel

$$= \frac{66.66}{1100} \times 100$$

$$= 6.06\%$$

4/1/2020

2) A post tensioned concrete beam 100mm wide, 300mm deep is Pre Stressed by 3 cables each with a c/s sectional area of 50mm² and with internal stresses of 1200 N/mm². All the 3 cables are straight and located 100mm from the soffit of the beam. If the modular ratio is 6. calculate the loss of stress in the 3 cables due to elastic deformation of concrete for only the following cases

a) simultaneously tensioned & anchoring of all the three Cables.

b) Successive tensioning of the 3 cables one at time.

Sol : Given data

wide(b) = 100mm

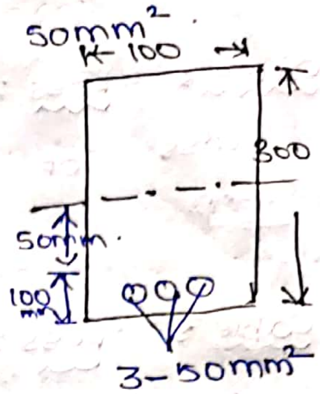
deep(d) = 300mm.

3 cables with c/s area 50mm².

Intial stress = 1200 N/mm².

modolus ratio (α_e) = 6. $\left(\frac{E_s}{E_c}\right)$.

eccentricity(e) = 50mm.



area of the beam = 100 * 300

= 3×10^4 mm².

Stress (F) = $\frac{\text{load}(P)}{\text{Area}}$

P = stress * Area

Area = ~~n×π~~ 50mm²

P = 1200 * 50

P = 60×10^3 N

$\boxed{P = 60\text{ KN}}$

Stresses at the level of steel (Fc) = $\frac{P}{A} + \frac{Pe^2}{I}$

$I = \frac{bd^3}{12} = \frac{(100)(300)^3}{12} = 225 \times 10^6\ \text{mm}^4$

$F_c = \frac{60 \times 10^3}{3 \times 10^4} + \frac{60 \times 10^3 \times (50)^2}{225 \times 10^6}$

$F_c = 2 + 0.66$

$\boxed{F_c = 2.66\ \text{N/mm}^2}$

a) Simentaneously tension — No losses.

b) Successive tension.

Cable-1 : Cable 1 is anchorage & tensioned & anchorage → No losses due to the elastic deformation.

cable-2 : Cable 2 is tensioned & anchorage → losses due to Cable-1.

Cable 3: Cable 3 is tensioned & anchoraged → loss due to cable 1 + cable 2.

Cable 2 Loss :

Loss of Stress in Cable-1 $= \alpha_e f_c$

$= 6(2.66)$

$= 15.96 \ N/mm^2$

Cable 3 Loss :

Loss of Stress in cable 1 $= \alpha_e * f_c = 6 * 2.66$

$= 15.96 \ N/mm^2$

Loss of stress in Cable 2 $= \alpha_e * f_c = 6 * 2.66$

$= 15.96 \ N/mm^2$

The total Loss of stress due to elastic deformation of Concrete in Cable 1 =

$= 15.96 + 15.96$

$= 31.92 \ N/mm^2$

Cable 2 $= 15.92 \ N/mm^2$

Cable 3 $= 0$

Average loss of Stress considering all the 3 Cable

$$\frac{31.92 + 15.92 + 0}{3} = 15.94 \ N/mm^2$$

It can be show that if the number of wires strands, bars are large. The loss due to elastic shortening does not exceed one half of the corresponding loss with Pre-tensioning.

Avg. Stress $\neq \frac{1}{2}(\alpha_e)(f_c)(n)$ n = no. of cable

$= \frac{1}{2}(6)(2.66)(3)$

$= 23.94 \ N/mm^2$

$15.94 \neq 23.94 \ N/mm^2$

Shrinkage :

3) A concrete beam is Prestressed by a Cable carrying an intial Prestressing force of 300 kN. The c/s area of wires in the cable is 300 mm². Calculate the Percentage loss of stress due to shrinkage of concrete using IS:1343- recomdations Assuming the beam to be a) Pretensioned b) Post tensioned

Assume ; $E_s = 210\ kN/mm^2$ & Age of concrete at transfer = 8 days.

Sol : Given data ;

$P = 300\ kN$

c/s area = 300 mm²

$t = 8$ days

a) Pre tensioned $\varepsilon_{cs} = 300 \times 10^{-6}$

$$\text{Loss of stress} = \varepsilon_{sc} \times E_s$$
$$= 300 \times 10^{-6} \times 210 \times 10^{+3}$$
$$= 63\ N/mm^2$$
$$= 0.063\ kN/mm^2.$$

b) post tensioned $= \dfrac{200 \times 10^{-6}}{\log_{10}(T+2)}$

$$= \frac{200 \times 10^{-6}}{\log_{10}(8+2)}$$
$$= 200 \times 10^{-6}$$

$$\text{Loss of stress} = \varepsilon_{cs} \times E_s$$
$$= 200 \times 10^{-6} \times 210$$
$$= 0.042\ N/mm^2.$$

$$\text{Intial stress} = \frac{300 \times 10^3}{300}$$
$$= 1\ kN/mm^2$$

Precentage loss for Pretensioned = $\frac{\ \ }{1} * 100$

= 6.3 %.

Pecentage loss for post tensioned = $\frac{0.042 \times 200 \times 10^{-6}}{1} * 100$

= 4.2 %.

Creep:

4) A post tensioned concrete beam rectangular Section 100mm wide & 300mm deep is Stressed by parabolic cables with 'zero' eccentricity at supports and an eccentricity of 50mm at the centre span. The area of cable is 200mm² and the intial stress of the cable is 1200 N/mm². If the ultimate creep strain is $30*10^{-6}$ mm/mm per N/mm² of stress and modolus of elasicity of Steel is 210kN/mm². Calculate the loss of stress in steel only due to creep of concrete.

Sol: Given data.

B = 300mm

d = 100mm.

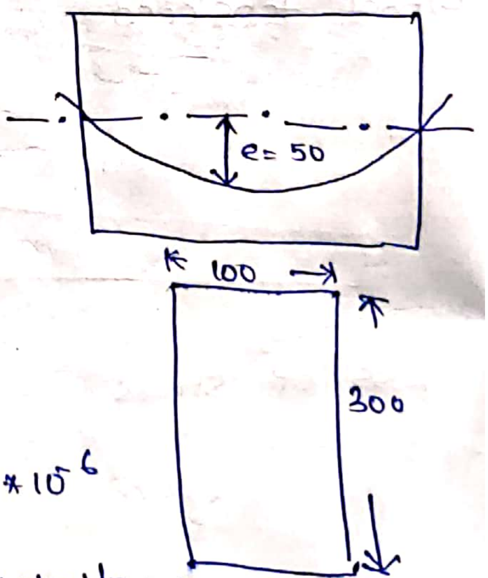
area = 200mm²

e = 50mm.

Stress = 1200 N/mm²

creep strain $\epsilon_{cc} = 30*10^{-6}$

E_s = 210kN/mm²



P = Stress * Area

= 1200 * 200

= 240 KN ⇒ $\boxed{P = 240*10^3 N}$

stress at level of steel $(F_c) = \frac{P}{A} + \frac{Pe^2}{I}$.

$I = \frac{bd^3}{12} = \frac{100(300)^3}{12}$

$\boxed{I = 225*10^6 \ mm^4}$

Parabolic $= \frac{2}{3}$

$A = 100*300 = 30*10^3 \ mm^2$

$F_c = \frac{240*10^3}{30*10^3} + \frac{2}{3}\left[\frac{240*10^3(50)^2}{225*10^6}\right]$

$= 8 + 2.66 \quad \simeq 9.77$

$\boxed{F_c = 10.66 \ N/mm^2}$

ultimate creep strain $= \varepsilon_{cc} * F_c * E_s$

$= 30*10^{-6} * 10.66 * 210*10^3$

$= 67.15 \ N/mm^2$

6-1-2020

Friction:

5). A concrete beam of 10m span, 100mm wide and 300mm deep is prestressed by 3cables. The area of each cable is 200mm² and the intial stress in the cable is 1200N/mm².

Cable-1 is parablic with an eccentricity of 50mm at the supports above the centroid. and the eccentricity is 50mm below the centre of span.

Cable-2 is also parabolic with zero eccentricity at supports and 50mm below the centroid.

Cable-3 is a straight with a uniform eccentricity 50mm below the centroid. If the cables are tensioned from one end only. Estimate the percentage loss of stress due to friction. Assume $\mu = 0.35$ & $K = 0.0015/m$. and eqn of parabola is written by

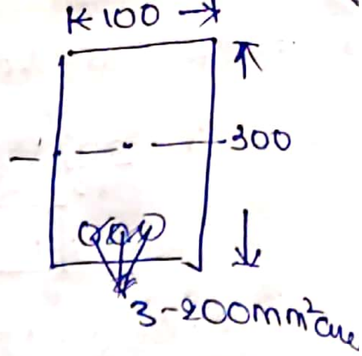
$y = \frac{4e}{L^2} * (L-x)$

Sol: Given data;

span (L) = 10m.

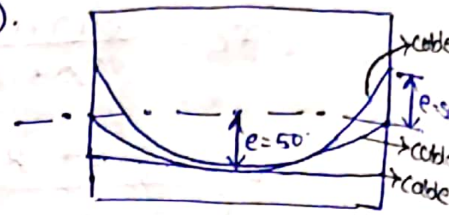
B = 100mm

D = 300mm



3 cables of 200mm² area (As).

Initial Stress = 1200 N/mm²



$\mu = 0.35$

$K = 0.0015/m.$

$$y = \frac{4e}{L^2} x [L-x]$$

$$\frac{dy}{dx} = \frac{4e}{L^2} (x) [L-x]$$

diff. the eqn.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{4e}{L^2}(x)(L-x)\right]$$

$$= \frac{d}{dx}\left[\frac{4e}{L^2}(Lx-x^2)\right]$$

at $x=0$; $\frac{dy}{dx} = \frac{4e}{L^2}(L-2(0))$

$$= \frac{4e}{L}.$$

Cable 1:

$$\frac{dy}{dx} = \frac{4e}{L}$$

$e_1 = 100mm$; $L = 10m = 10*10^3 m$

$$\frac{dy}{dx} = \frac{4(100)}{10(10^3)} = 0.04 \text{ rad}$$

α = total slope = $2 \times (0.04) = 0.08$ rad

Cable 2:

$$\frac{dy}{dx} = \frac{4e}{L}$$

$e_2 = 50mm$; $L = 10m$

$$\frac{dy}{dx} = \frac{4(50)}{10(10^3)} = 0.02\,rad$$

α = total slope = 2(0.02) = 0.04 rad

cable 3:

$$\frac{dy}{dx} = \frac{4e}{L}$$

e = 0mm; L = 10m

$\frac{dy}{dx} = 0$; $\alpha = 0$

Po = intial stress * Area of cable

= 1200 (200) = 2400 kN

$\boxed{P = 240\,kN}$

Cable 1: $P_x = P_0 [\mu\alpha + kx]$ (x = 10)

= P_0 [0.35 * 0.8 + 0.0015 * 10] ⇒ P_0[0.043]

Cable 2: Loss of stress. $P_x = P_0(\mu\alpha + kx)$

= P_0[0.35 * 0.04 + 0.0015 * 10]

= P_0[0.029]

cable 3: $P_x = P_0(\mu\alpha + kx)$

= P_0[0.35 * 0 + 0.0015 * 10]

= P_0[0.015]

Cable NO	loss of stres	% loss of stress.
1	1200 * 0.043 = 51.6	$\frac{51.6}{1200}$ * 100 = 4.3%.
2	1200 * 0.029 = 34.8	$\frac{34.8}{1200}$ * 100 = 2.9%.
3.	1200 * 0.015 = 18	$\frac{18}{1200}$ * 100 = 1.5%.

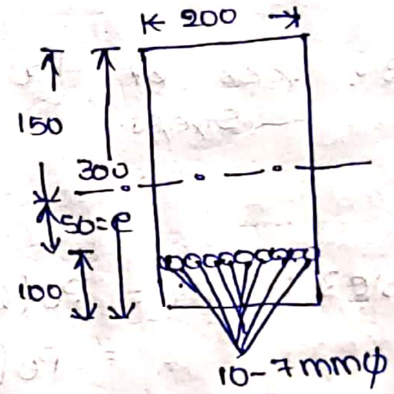
Total losses:

6) A Pretensioning beam 200mm wide and 300mm deep is Prestressed by 10wires of 7mm diameter intially Stressed to 1200N/mm² with their centroids located 100mm from the soffit. Find the maximum stresses in concrete immediately after transfer, Allowing only for elastic shortening (deformation) of concrete

If the concrete undergoes further shortening due creep & shrinkage while their is a relaxation of 5% of steel stress. Estimate the final Percentage of loss of stress in the wires using IS:1343 recomdations and the following data. $E_s = 210\ kN/mm^2$, $E_c = 5700\sqrt{F_{cu}}$, $f_{cu} = 42\ N/mm^2$; creep coefficient $(\phi) = 1.6$ the total residual shrinkage strain $= 3 \times 10^{-4}$ (ε_{cs}).

7-1-2020

Sol: Given data,



$B = 200mm$

$D = 300mm$.

No. of bars $= 10 - 7mm\phi$.

$e = 50mm$

Stress $= 1200 N/mm^2$

$f = \frac{P}{A}$.

$P = F * A = 1200 * 10 \cdot \frac{\pi}{4}(7)^2$ $\quad n \cdot \frac{\pi}{4} d^2$ $= 384.84$

$P = 461.81 * 10^3 N$

$\boxed{P = 461.81 kN}$

$f_c = 42 N/mm^2$

$E_c = 5700\sqrt{F_{cu}}$

$= 5700\sqrt{42}$

$\boxed{E_c = 36.94\ kN/mm^2}$

$\varepsilon_{cs} = 3 \times 10^{-4}$ (Pre tensioning)

$F_c = \frac{P}{A} + \frac{Pe^2}{I}$

$I = \frac{bd^3}{12} = \frac{(200)(300)^3}{12} = 450 \times 10^6 \ mm^4$

$F_c = \frac{461.81 \times 10^3}{60 \times 10^3} + \frac{461.81 \times 10^3 (50)^2}{450 \times 10^6}$

$= 7.69 + 2.56$

$\boxed{F_c = 10.25 \ N/mm^2}$

Elastic deformation $= \alpha_e f_c$

$= \left(\frac{E_s}{E_c}\right) f_c$

$= \left(\frac{210}{36.44}\right) 10.25 \times 10^{-3}$

$= 58.27 \ N/mm^2$

force in wires @ immediately after trafer

Force = Stress * Area As (wires)

$= [1200 - 58.27][384.84]$

$\boxed{P = 439.38 \ KN}$

$F_c = \frac{P}{A} + \frac{Pe^2}{I}$

$= \frac{439.38 \times 10^3}{60 \times 10^3} + \frac{439.38 \times 10^3 \times (50)^2}{450 \times 10^6}$

$= 7.32 + 2.441$

$\boxed{F_c = 9.76 \ N/mm^2}$

Total losses :

i) Elastic deformation :-

$\alpha_e f_c$

$\left(\frac{E_s}{E_c}\right) * f_c$

$$=\left(\frac{210}{36.94}\right)\times 10.25\times 10^{-3}$$

$$= 58.27\ N/mm^2$$

Creep:

$$\varepsilon_c = \phi\, \alpha_e\, f_c$$

$$= 1.6\left(\frac{210}{36.94}\right)(9.76)\times 10^{3}) .$$ 5.68.

$$= 88.77\ N/mm^2$$

Shrinkage:

$$\varepsilon_s = \varepsilon_{cs} \times E_s$$

$$= 3\times 10^{-4} \times 210\times 10^{3}$$

$$= 63\ N/mm^2$$

Relaxation:

Relaxation of 5% steel $= \frac{5}{100}(1200)$

$= 60\ N/mm^2$

Total losses in steel $= 58.27+88.77+63+60$

$= 270.04\ N/mm^2$ ✓

Remaining stresses in wires: = Intial stress − final stress

$= 1200 - 270.04$

$= 929.96\ N/mm^2$

% loss of stress $= \frac{270.04}{1200}\times 100$

$= 22.50\%$.

8-1-2020

Anchorage slip

7) A posttensioned cable of beam 10m long is initially tensioned to a stress of 1000N/mm² at one end. If the tendons are curved so that the slope is 1 in 24 at each end with an area of 600mm², calculate the loss of prestress due to friction given the following data:-

→ Coefficient of friction(μ) b/w duct & cable = 0.55

→ friction coefficient for wave effect(k) = 0.0015/m.

→ During anchoring if there is a slip of 3mm at the jacking end(Δ). Calculate the final force in the cable & percentage loss of prestress due to friction and slip. $E_s = 210\ kN/mm^2$

Sol:- Given data,

Length (l) = 10m.

Stress (f) = 1000N/mm² at one end.

Slope = 1 in 24

Area(A) = 600mm².

$\mu = 0.55$

$k = 0.0015/m$

$\Delta = 3mm$

$E_s = 210\ kN/mm^2$.

$P = F \times A$

$= 1000 \times 600$

$\boxed{P_0 = 600\ kN}$.

Friction :-

$P_x = P_0(\mu\alpha + kx)$. (tendons > 1)

$\alpha = 2\left(\frac{1}{24}\right)$

$\alpha = \frac{1}{12}$

$P_x = 1600 \times 10^3 \times \left[0.55\left(\frac{1}{12}\right) + 0.0015\ \overset{l}{(10)}\right]$

$\boxed{P_c = 60.83 kN/mm^2}$

Loss due to anchorage slip:

$$\Delta = \frac{PL}{A E_s}$$

$$P = \frac{\Delta A E_s}{L}$$

$$= \frac{3(600)(210 * 10^3)}{10 * 10^3}$$

$$\boxed{P = 37.8 kN}$$

Loss of force due to friction:

$$f = P_c * A$$

$$= 60.83 * 600$$

$$\boxed{F = 36.94 \ kN}$$

Total loss of force due to friction & slip

$$= 37.8 + 36.94$$

$$\boxed{f = 74.74 kN}$$

$$A - P_0$$

Final force in cable $= 600 - 37.74$

$= 525.26$ kN

% Loss of Prestress $= \frac{74.74}{600} * 100 = \frac{P}{A} *$

$= 12.45$ %